Neuro-Economic Modeling of Liquid Satiation and Utility Optimization: A Theoretical and Simulation-Based Analysis

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Abstract

This study develops a theoretical and simulation-based framework for examining diminishing marginal utility and satiation in divisible liquid consumption. The model connects classical consumer theory with findings from neuroeconomics while keeping economic marginal utility, subjective valuation, and dopamine reward-prediction error analytically distinct. A linear marginal-utility specification, $MU(q) = \alpha - \beta q$, is used to derive a concave total-utility function and a finite model-implied satiation quantity. To demonstrate the econometric procedure, a synthetic dataset of 150 simulated decision-makers, each observed at six quantity levels, is generated under a prespecified data-generating process. The parameters are then recovered by ordinary least squares (OLS), and then functional-form, subsample, and counterfactual-price analyses are performed. The simulated results indicate that a positive parameter for a marginal-utility decline is recovered close to the data-generating $\beta = 2$, and that the model-implied satiation point is close to six units. Counterfactual calculations show that the model predicts lower optimal consumption as price rises. Because the dataset is simulated rather than collected from human participants, the results are not evidence about actual human consumption, physiological satiation, or neural activity. The paper's contribution is therefore methodological: it demonstrates how a utility model can be formalized, estimated, stress-tested, and connected cautiously to neuroeconomic theory without treating neural signals as direct measures of economic utility.[1]

Keywords: neuroeconomics; dopamine; reward-prediction error; diminishing marginal utility; law of demand; divisible liquid consumption; OLS simulation; valuation

Introduction:

Consumer theory commonly represents choice as the maximization of utility subject to constraints. One classical implication is diminishing marginal utility: as consumption of a single good increases, the additional benefit from another unit generally falls. Neuroeconomics provides a complementary biological perspective by studying neural mechanisms involved in valuation, learning, and reward prediction. Research on dopamine reward-prediction errors (RPEs) has shown that neural responses can encode deviations between expected and received rewards, while

[1] All numerical observations in this manuscript are simulated. No human participants were recruited and no human-subject data were collected.

broader neuroeconomic frameworks examine how neural signals relate to value-based choice (Schultz et al., 1997; Rangel et al., 2008; Schultz, 2016).

These literatures are related but should not be treated as equivalent. Economic marginal utility is a construct describing the change in utility associated with an additional unit of a good. A subjective hedonic rating is a reported psychological response. A dopamine RPE is a neural signal associated with prediction error. Evidence that neural responses correlate with reward or valuation therefore does not, by itself, establish that a measured dopamine signal is an economic marginal utility. The present study adopts this distinction explicitly and uses neuroeconomic findings only as theoretical motivation.

The study asks two questions: how does a simple diminishing-marginal-utility model generate a finite satiation quantity when consumption is divisible, and how stable are the model's parameters and predictions under alternative specifications, sample sizes, and prices? This paper investigates these questions through a transparent simulation rather than a human experiment.

Research Questions:

RQ_1: How does a linear diminishing-marginal-utility function produce a finite satiation quantity and an interior consumption optimum?

RQ_2: How accurately can ordinary least squares (OLS) estimate the parameters of the prespecified model using simulated observations, and how do its predictions change across different model specifications and price scenarios?

Hypotheses and Model Implications:

H_1 (estimable hypothesis): The simulated marginal utility decreases with quantity, implying a negative OLS coefficient on q. Equivalently, the function $MU(q) = \alpha - \beta q$ has a positive decline parameter precisely when $\beta > 0$.

H_2 (model implication): If $\beta > 0$, the total-utility function is strictly concave and reaches its interior maximum at $q^* = \alpha/\beta$. This is a mathematical implication of the model rather than an independently observed behavioral outcome.

H_3 (counterfactual implication): Under $\max_q[U(q) - pq]$, the model implies $q(p) = (\alpha - p)/\beta$ when $0 \leq p < \alpha$, so higher positive prices reduce the model's optimal quantity. This is evaluated as a counterfactual simulation, not observed price elasticity.

Literature Review:

Early marginalist economics sought to formalize how people evaluate goods and choose among alternatives. During the 1870s, Jevons, Menger, and Walras recast the informal idea of usefulness through the concept of marginal utility. Von Neumann and Morgenstern later extended this line of work in their 1944 study, establishing a formal account of utility under uncertainty. That earlier work matters for the present model because it shows that the value assigned to one additional unit of a good need not stay constant as consumption increases. At the same time, the connection between diminishing marginal utility and observed demand is not as straightforward as a simple textbook diagram might suggest. Beattie and LaFrance (2006), for example, reconsidered the relationship between the law of demand and diminishing marginal utility, showing that the relationship can depend on income effects and the structure of preferences. The economic concept of marginal utility is therefore best treated as a theoretical construct rather than as a direct physiological measurement.

The development of neuroeconomics provided another way of thinking about value. Rangel et al. (2008) developed a framework for studying the neural and computational basis of value-based decision making, bringing together economics, psychology, and neuroscience. Their framework helped establish a common language for discussing valuation, learning, and choice across these disciplines. This is relevant to the present study because the model also connects an economic representation of marginal utility with a biological literature on reward, but it does not assume that the two are literally identical. Schultz et al. (1997) had earlier linked dopamine activity to reward-prediction errors, while Steinberg et al. (2013) provided causal evidence that dopamine neurons can influence learning from prediction errors. Stauffer et al. (2016) are particularly relevant here because their experimental work connected dopamine reward-prediction-error responses with marginal utility. Their findings provide a useful bridge between the economic concept used in the present model and neural evidence, even though the two should not be treated as the same measurement.

Watabe-Uchida et al. (2017) subsequently reviewed the neural circuitry involved in reward-prediction-error processing and showed that these processes involve distributed pathways rather than a single centre responsible for pleasure or utility. The literature after 2017 has continued to complicate the simple interpretation of dopamine as a single-value signal. Dabney et al. (2020), for example, proposed a distributional account of dopamine coding in which different dopamine neurons may represent different parts of a reward distribution rather than one single scalar estimate of value. Their findings are relevant to the present framework because they suggest that reward valuation can contain more information than a single numerical value. The economic marginal-utility function used here is intentionally much simpler, but the newer neuroscience literature provides a reason to treat that simplicity as a modelling assumption rather than as a literal description of neural processing.

Research on satiation has also moved beyond the idea that consumption simply delivers a fixed reward. Lasschuijt et al. (2021) reviewed the neurophysiological mechanisms through which oro-sensory exposure contributes to satiation and food intake. Their review discusses how sensory exposure and signals from the gastrointestinal system are integrated with brain processes involved in reward and appetite. This is particularly relevant to a model of liquid consumption because the quantity consumed is not necessarily independent of the sensory experience associated with that consumption. In practical terms, drinking another glass is not just "one more unit" in the same psychological sense as the first glass. The experience, internal state, and physiological consequences may all change along the way.

More direct evidence for this point comes from Grove et al. (2022), who examined dopamine subsystems involved in food and water consumption. They found that different dopamine neurons responded to different stages and consequences of ingestion, including changes in hydration and signals associated with nutrients. Their findings suggest that the value of consuming water or food can be related to internal physiological states rather than only to the immediate sensory reward. As Grove et al. (2022, p. 374) put it, "Food and water are rewarding in part because they satisfy our internal needs." This finding is useful for the present paper, although it also places a clear limit on the interpretation of the simulation. The six-unit optimum generated by the model cannot be treated as a physiological satiation threshold because the simulation contains no measurements of hydration, gastrointestinal feedback, or neural activity.

Another recent line of work has compared different kinds of rewards rather than assuming that all rewards are represented in exactly the same way. Oren et al. (2022) examined neural responses to food and monetary reward delivery using fMRI and parametrically varied the subjective value of both reward types. Their work suggests that food and money can engage partly shared value-related processes while also retaining differences related to the biological and learned nature of the rewards. For the present study, this distinction matters because liquid consumption is a biologically grounded form of reward. Its value may depend on sensory and physiological factors that would not be captured by a purely abstract monetary utility model.

The broader field of decision neuroscience has also become more cautious about reducing value-based choice to one neural mechanism. Dennison, Sazhin, and Smith (2022) reviewed advances in decision neuroscience and neuroeconomics, focusing on reward learning, valuation, risk, ambiguity, intertemporal choice, emotion, and individual differences. Their analysis points to substantial progress, while also showing that several conceptual and methodological questions remain unresolved. That caution fits the present study. An OLS coefficient generated from simulated marginal utility should not be presented as if it were a neural measurement. At most, the coefficient provides a formal representation of declining modeled value that can be discussed alongside, rather than equated with, neuroeconomic evidence.

More recent work has continued to question whether dopamine prediction errors should be understood only as signals about a single integrated value. Kahnt and Schoenbaum (2025), for example, reviewed evidence suggesting that dopaminergic prediction-error signals may carry information about reward features beyond integrated value. Their discussion is useful because it reinforces a point already emerging from the earlier literature: dopamine signalling is richer and more context-dependent than a simple one-to-one translation from reward to utility. A 2026 review by Dudhabhate and Costa similarly traces the development of the dopamine prediction-error account and notes that, although the RPE framework remains influential, its interpretation continues to develop. These newer contributions make it harder to justify a direct equivalence between dopamine activity and economic marginal utility, but they also make the theoretical connection more interesting.

Taken together, research since the 2000s indicates that consumption and valuation depend on several interacting processes. Dopamine activity conveys information about predicted rewards and value. At the same time, consuming food and fluids produces sensory and physiological feedback, changing the organism's internal state. The relationship is therefore more complicated than a direct equation between dopamine activity and economic utility. This is especially relevant to the present model of liquid consumption, where marginal utility is assumed to decline smoothly as quantity rises. The assumption is mathematically convenient and economically familiar, but the biological literature suggests that actual consumption may involve thresholds, changing internal states, sensory adaptation, and other nonlinear processes.

Within the literature reviewed for this study, relatively little work appears to combine a simple diminishing-marginal-utility model for divisible liquid consumption with a reproducible OLS simulation and a deliberately cautious neuroeconomic interpretation. Existing economic, behavioural, and neurophysiological studies generally examine these questions separately, using either theoretical models, human or animal experiments, neuroimaging, or physiological measurements. The present paper therefore makes a modest methodological contribution rather than claiming strong substantive novelty. It demonstrates one transparent way to place the classical utility model alongside contemporary research on dopamine, reward, valuation, and satiation while keeping the simulated econometric results separate from claims about actual human behaviour or neural activity.

Theoretical Framework:

The model is initiated with a linear marginal-utility function expressed as follows:

$$MU(q) = \alpha - \beta q, \quad \alpha > 0, \beta > 0$$

Here, α is the marginal utility when quantity (q) equals zero, and β measures how quickly marginal utility falls as q rises. Integrating marginal utility over the interval from zero to q gives total utility as $U(q) = \alpha q - (\beta/2)q^2$. Setting U(0) equal to zero makes the constant C equal to zero. To maximize total utility, set the first derivative equal to zero: $\alpha - \beta q = 0$. This yields $q = \alpha/\beta$. Since the second derivative is $-\beta < 0$, this point is a strict maximum within the continuous model. Price-Constrained Optimization: Given a unit price p, the consumer maximizes the modeled net benefit, $\max_q[U(q) - pq]$. The first-order condition, $\alpha - \beta q - p = 0$, implies that quantity demanded is $q(p) = (\alpha - p)/\beta$ when $0 \leq p < \alpha$. Once p reaches α or higher, the solution is at the corner, with $q(p) = 0$. This theoretical demand function comes from the model; it is not an empirical estimate of real-world price elasticity.

Measurement Clarification: This study uses simulated observations[2] and does not claim to measure human utility, cardinal utility, hedonic experience, or neural activity. Here, marginal utility (MU) refers to a continuous outcome variable generated by the model. No subjective 0–10 rating is converted into cardinal utility, and no neural measurement is inferred from the simulated data. This avoids treating an ordinal self-report scale as cardinal utility and keeps the interpretation of regression coefficients within the assumptions of the simulation.

Methodology:

Simulation Design: We generate a reproducible synthetic dataset for 150 simulated decision-makers, observing each one at six sequential quantity levels (q = 1,...,6), for producing 900 observations in total. The data-generating process is $MU_{iq} = 12 - 2q_{iq} + \varepsilon_{iq}$. Here, ε_{iq} denotes a mean-zero random disturbance. The disturbance standard deviation increases modestly with q. A fixed random seed is used so that the simulation can be reproduced. The disturbance standard deviation increases modestly with q. A fixed random seed is used so that the simulation can be reproduced.

The 150 simulated decision-makers were not human participants. No individuals were recruited; no juice was consumed, questionnaires administered, or physiological or neural measurements obtained. The simulation is solely an analytical demonstration.

[2] **Reproducibility Statement:** All observations are simulated. The simulation uses random seed 20260821, 150 simulated decision-makers, six quantity levels per decision-maker, and the data-generating process $MU = 12 - 2q + \varepsilon$. The purpose is to make clear that the reported numerical results are illustrative outputs of a computational model and are not observations from human participants.

Econometric Specification: For the simulated data, the estimated regression is MUiq = α + γqiq + εiq, with γ representing the ordinary least squares (OLS) slope obtained directly from the regression. Since β is defined as the positive decline parameter, β = −γ. The estimate of $\hat{\beta} = -\Sigma(qi-\bar{q})(MUi-\bar{MU})/\Sigma(qi-\bar{q})^2$, estimate of α as $\hat{\alpha} = \bar{MU} + \hat{\beta}\bar{q}$. This notation corrects the sign inconsistency in the earlier manuscript.

Robustness Procedures: The analysis reports descriptive patterns, OLS parameter recovery, goodness of fit, a quadratic functional-form check, repeated half-sample estimation, and counterfactual demand calculations under alternative prices. These analyses test the behavior and recoverability of the specified model; they do not establish empirical facts about human consumers.

Simulation Results:

Table 1: Descriptive Pattern

q	Theoretical MU	Simulated Mean MU
1	10.00	10.00
2	8.00	7.98
3	6.00	6.00
4	4.00	3.99
5	2.00	2.00
6	0.00	-0.00

The simulated means follow the intended downward pattern. The data-generating process was deliberately specified with α = 12 and β = 2. Estimates close to those values are therefore expected; they should not be interpreted as evidence about actual consumer behavior.

Table 2: OLS Parameter Recovery

Parameter	Estimate	SE
$\hat{\alpha}$	11.993	—
$\hat{\gamma}$	-1.999	0.003
$\hat{\beta} = -\hat{\gamma}$	1.999	0.003

In the simulation, the estimated parameters were $\hat{\alpha} = 11.993$ and $\hat{\beta} = 1.999$. The negative slope was also statistically significantly different from zero in the simulated sample ($t = -579.80$, $p < .001$). This demonstrates estimator recovery under the specified data-generating process.

Table 3: Goodness of Fit

Measure	Value
R^2	0.997
Observations	900
Residual SSE	28.024

The high R^2 is a feature of the intentionally simple simulation: marginal utility is generated directly from a linear function of quantity with relatively small random disturbances. It should not be interpreted as evidence that actual human utility is almost perfectly linear.

Model Implications: The estimated satiation point is $q^* = \hat{\alpha}/\hat{\beta} = 6.00$ units. Because this follows directly from the estimated functional form, it is a model-implied optimum rather than an independently observed physiological satiation threshold. H3 is likewise an algebraic implication: for $0 < p < \alpha$, $q(p)$ is lower than $q(0)$. It is therefore presented as a counterfactual model result, not as an empirical price response.

Robustness and Counterfactual Analysis:

Alternative Functional Form: As a specification check, a quadratic model was estimated: $MU = \alpha_0 + \gamma_1 q + \gamma_2 q^2$. It produced coefficient estimates of $\alpha_0 = 11.998$, $\gamma_1 = -2.002$, and $\gamma_2 = 0.0005$. The implied turning point is approximately nan units. Because both specifications use the same simulated data, this demonstrates functional-form sensitivity rather than empirical robustness.

Table 4: Half-Sample Stability

Parameter	Mean	SD	Min	Max
$\hat{\alpha}$	11.992	0.012	11.973	12.015
$\hat{\beta}$	1.999	0.004	1.994	2.005

Parameter estimates remain near the data-generating values across repeated half-sample draws. This indicates predictable estimator behavior under the simulation design, not general evidence about real consumers.

Table 5: Counterfactual Demand Analysis Under Alternative Prices

Price p	Model-implied q(p)	Change from p=0
0	6.00	0.0%
1	5.50	-8.3%
2	5.00	-16.7%
4	4.00	-33.4%
6	3.00	-50.0%
8	2.00	-66.7%
10	1.00	-83.4%
12	0.00	-100.0%

The counterfactual results show the expected downward relationship between price and modeled quantity. Since price is varied algebraically rather than experimentally, this is not an estimate of observed price elasticity.

Heterogeneity: The earlier manuscript included subgroup estimates implying socio-demographic heterogeneity. Because this simulation contains no real participants or demographic observations, those estimates have been removed rather than fabricated. Subgroup analysis should be added only when actual subgroup data are available.

Discussion:

The simulation demonstrates that a simple linear marginal-utility model can be estimated using OLS and used to derive a finite model-implied utility maximum and a price-constrained optimum. Parameter recovery works as intended because the simulated data-generating process is deliberately aligned with the theoretical specification.

The neuroeconomic interpretation remains deliberately cautious. Dopamine RPE research provides evidence about neural responses to prediction errors and reward-related information, but this simulation observes no dopamine activity. Nor does it establish that economic MU is biologically identical to an RPE. Neuroeconomic research therefore motivates the question of valuation without serving as direct physiological validation of the estimated coefficient.

The price schedule should likewise be interpreted as a model-generated counterfactual. Changing p in the optimization equation while holding $\hat{\alpha}$ and $\hat{\beta}$ fixed demonstrates the model's comparative statics; it does not identify a behavioral demand curve.

Implications:

The primary implication is methodological. The framework shows how diminishing marginal utility, optimization, and econometric estimation can be integrated in a transparent simulation while maintaining a clear distinction between simulated evidence and human evidence. Future empirical work could test these predictions with observed consumption, prices, subjective valuation measures, and, where appropriate, physiological data.

Limitations and Future Research:

The central limitation is that all observations are simulated. Consequently, the results provide no empirical evidence about actual consumption, physiological satiation, or neural activity. The data-generating process is intentionally close to the estimated model, making parameter recovery unsurprising. The continuous quantity framework sets aside indivisibilities, substitution among multiple goods, real budget constraints, measurement error, and differences in preferences. It also makes no distinction between physiological satiation and the psychological processes behind price is not experimentally manipulated, so the counterfactual price results are theoretical comparative statics. Finally, no dopamine or other physiological measure is collected. Future research could address these limitations through a preregistered human-subject study, validated valuation measures, observed or randomized prices, and appropriate physiological or neural measures.

Conclusion:

This paper presents a theoretical and simulation-based model of diminishing marginal utility and liquid-consumption satiation. A reproducible synthetic dataset demonstrates how OLS can recover a positive decline parameter, how that parameter generates a finite model-implied satiation point, and how alternative prices change modeled optimal quantity. The study does not claim that these simulated estimates describe actual consumers or physiological processes. Its contribution is a transparent analytical framework connecting classical utility theory, econometric estimation, and cautious neuroeconomic interpretation. Future empirical research can test whether the model's predictions survive when actual behavioral, price, and physiological data are introduced.

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